

Review of Rational Expressions and Equations

A rational expression is an expression containing fractions where the numerator and/or denominator may contain algebraic terms.

1. Simplify $\frac{6}{14}$.

Identification/Analysis	Identify this as an expression . Notice it has only one term , so simplify means be sure the numerator and denominator do not have any common factors. In other words, we need to make sure the greatest common factor (GCF) is a 1.	
Solution	$6 = 2 * 3$ and $14 = 2 * 7$ The GCF is 2.	Factor numerator and denominator. Identify GCF.
	$\frac{2*3}{2*7} = \frac{2}{2} * \frac{3}{7} = 1 * \frac{3}{7} = \frac{3}{7}$	Factor the GCF out of both the numerator and denominator. Reduce. Expression simplified.
Check/Verification	Since 3 and 7 have a GCF of 1, we know the fraction is simplified.	

2. Simplify $\frac{5}{6} + \frac{7}{20}$.

Identification/Analysis	Identify this as an expression . Simplify means combine the two terms into a single term by adding fractions. Recall to add fractions, either we have common denominators or we must write equivalent fractions that have a common denominator. In this case the denominators are not the same, therefore factor each denominator into a product of prime numbers and determine the least common denominator (LCD) .	
Solution	$6 = 2 * 3$ and $20 = 2 * 2 * 5$ The LCD is $2 * 2 * 3 * 5$ or 60	Factor denominators. Find LCD.
	$\frac{5}{6} = \frac{(10)5}{(10)6} = \frac{50}{60}$ $\text{and } \frac{7}{20} = \frac{(3)7}{(3)20} = \frac{21}{60}$ <p>Now $\frac{5}{6} + \frac{7}{20}$ can be written as $\frac{50}{60} + \frac{21}{60}$</p> $\frac{50}{60} + \frac{21}{60} = \frac{71}{60}$	Write equivalent fractions with LCD. Add fractions together. Expression simplified.
Check/Verification	Check to see if the fraction can be simplified, that is, if there is a common factor in both the numerator and denominator. Since the GCF of 71 and 60 is 1, the fraction is simplified so we are finished.	

3. Simplify $\frac{5}{12x} + \frac{1}{8x^2}$.

Identification/Analysis	This is an expression with two terms. Simplify means combine the two terms into a single term by adding fractions. In order to add fractions, you must have a common denominator. Therefore my first step is to find the LCD.	
Solution	$12x = 2 * 2 * 3 * x$ and $8x^2 = 2 * 2 * 2 * x * x$ The LCD is $2 * 2 * 2 * 3 * x * x$ or $24x^2$	Factor denominators Find LCD
	$\frac{5}{12x} = \frac{(2x)5}{(2x)12x} = \frac{10x}{24x^2}$ and $\frac{1}{8x^2} = \frac{(3)1}{(3)8x^2} = \frac{3}{24x^2}$ $\frac{5}{12x} + \frac{1}{8x^2} = \frac{10x}{24x^2} + \frac{3}{24x^2} = \frac{10x+3}{24x^2}$	Write equivalent fractions with LCD. Since they have common denominators, add the numerators. Expression simplified.
Check/Verification	The numerator is simplified because the two terms are not like terms. The fraction is simplified because the GCF of the numerator and denominator is 1.	

4. Simplify $\frac{x+3}{x-2} - \frac{4x+7}{x^2+3x-10}$.

Identification/Analysis	This is an expression with two terms. Simplify means combine the two terms into a single term by subtracting the fractions. To subtract fractions, you must have a common denominator. Therefore the first step is to find the LCD. Notice the denominators are different than the previous examples. $x-2$ is a two-term polynomial , called a binomial and $x^2+3x-10$ is a three-term polynomial called a trinomial. To factor these denominators, we must remember how to factor polynomials .	
Solution	$x-2$ $x^2+3x-10$ $-1*10$ and $-2*5$ are factors of -10 . Also, $-2+5=3$. $x^2+3x-10=(x+5)(x-2)$	Factor each denominator. GCF of the two terms is 1. It is not a difference of squares. Prime (will not factor further) GCF of the three terms is 1. Quadratic of form x^2+bx+c . Find factors that multiply to -10 and add to 3.
	The most number of $x-2$ factors in either expression is one. The most number of $x+5$ factors in either expression is one. Therefore the LCD is $(x+5)(x-2)$.	Find the LCD of $x-2$ and $(x+5)(x-2)$. Determine the most number of each factor in either denominator.
	$\frac{x+3}{x-2} = \frac{(x+5)(x+3)}{(x+5)(x-2)} = \frac{x^2+8x+15}{(x+5)(x-2)}$ $\frac{4x+7}{x^2+3x-10} = \frac{4x+7}{(x+5)(x-2)}$ $\frac{x+3}{x-2} - \frac{4x+7}{x^2+3x-10} = \frac{x^2+8x+15}{(x+5)(x-2)} - \frac{4x+7}{(x+5)(x-2)}$ $= \frac{x^2+8x+15-(4x+7)}{(x+5)(x-2)}$ $= \frac{x^2+4x+8}{(x+5)(x-2)}$	Write equivalent fractions with the LCD This fraction already has the LCD, so no equivalent fraction is needed. Write both fractions with a common denominator. Subtract. Be careful with the minus sign! Combine like terms.

Check/Verification	Finally, we need to ensure the fraction $\frac{x^2 + 4x + 8}{(x + 5)(x - 2)}$ is simplified. That means make sure the GCF of the numerator and denominator is 1. Therefore, try to factor the numerator. Since there is no pair of numbers that both multiply to 8 and add to 4, the numerator does not factor. Therefore the GCF is 1, and we are finished.
--------------------	--

Review of Rational Equations:

Equations are fundamentally different than expressions both in appearance and in purpose. First, equations include an equal sign. Second, an equation is a statement about the equality of two expressions. Typically your job is to determine the value or values for the variable which make the equation true.

By definition, rational equations include fractions, often with variables in the denominator. It is important to remember that division by zero is not defined. Therefore one of the first things to do is consider what values would make the denominator zero, and thus make the fraction undefined. The values that make the denominator to equal zero are not in the domain.

The general strategy for solving a rational equation is to find the LCM of all the denominators and multiply the entire equation by this value. This will eliminate all the denominators and leave you with an equation that will be easier to solve. Frequently the resulting equation is either linear or quadratic, though other possibilities exist. The following examples will illustrate this general strategy.

1. Solve the equation $\frac{1}{2} + \frac{x}{8} = 3$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction. Clear the fractions by multiplying both sides of the equation by the LCD.	
Solution	There are no variables in the denominator, so the domain includes all real numbers.	Find the domain.
	2 is prime and $8 = 2 * 2 * 2$ The LCD is $2 * 2 * 2$ or 8	Find the LCD. Factor all the denominators.
	$8\left(\frac{1}{2} + \frac{x}{8}\right) = 8 * 3$ $8\left(\frac{1}{2}\right) + 8\left(\frac{x}{8}\right) = 8(3)$ $4 + x = 24$ $x = 20$	Solve. Multiply both sides of the equation by the LCD to eliminate (reduce) the fractions. The resulting equation is linear. Solve for x .
Check/Verification	$\frac{1}{2} + \frac{20}{8} = 3$ $\frac{1}{2} + \frac{5}{2} = 3$ $3 = 3$	Substitute $x = 20$ into the original equation and simplify the results. The statement is true so $x = 20$ is a solution.

2. Solve $\frac{5}{12x} + \frac{1}{8x^2} = \frac{3}{x}$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction, so clear the fractions by multiplying both sides of the equation by the LCD. Since the denominator contains a variable, determine the domain restrictions.	
Solution	$12x = 0$ if and only if $x = 0$. $8x^2 = 0$ if and only if $x = 0$. $x = 0$ if and only if $x = 0$. Therefore $x \neq 0$. The domain does not include zero. Zero cannot be a solution to the equation.	Find the domain restrictions. Set denominators with variables equal to zero.
	$12x = 2 * 2 * 3 * x$ $8x^2 = 2 * 2 * 2 * x * x$ $x = x$	Factor each denominator.
	LCD is $2 * 2 * 2 * 3 * x * x = 24x^2$	Find the LCD. Build the LCD using the factors above.
	$24x^2 \left(\frac{5}{12x} + \frac{1}{8x^2} \right) = 24x^2 * \frac{3}{x}$ $24x^2 \left(\frac{5}{12x} \right) + 24x^2 \left(\frac{1}{8x^2} \right) = 24x^2 \left(\frac{3}{x} \right)$ $(2x)(5) + (3)(1) = (24x)(3)$ $10x + 3 = 72x$ $3 = 62x$ $x = \frac{3}{62}$	Multiply both sides of the equation by LCD to eliminate (reduce) the fractions. Distribute the LCD to both terms on the left side. Simplify the fractions. This is a linear equation. Solve for x . The solution is in the domain.
Check/Verification	Substitute $x = \frac{3}{62}$ into the original equation and simplify the results to verify it is a solution.	

3. Solve $\frac{5}{12x} + \frac{1}{8x^2} = \frac{1}{3}$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction, so clear the fractions by multiplying both sides of the equation by the LCD. Since the denominator contains a variable, determine the domain restrictions.	
Solution	$12x = 0$ if and only if $x = 0$. $8x^2 = 0$ if and only if $x = 0$. 3 has no variable factor, so the denominator cannot equal 0 . Therefore $x \neq 0$. The domain does not include zero. Zero cannot be a solution to the equation.	Find the domain. Set denominators with variables equal to zero. Solve for x . State the restrictions for the domain.
	$12x = 2 * 2 * 3 * x$ $8x^2 = 2 * 2 * 2 * x * x$ $3 = 3$	Factor each denominator.
	$12x = 2 * 2 * 3 * x$ $8x^2 = 2 * 2 * 2 * x * x$ $3 = 3$ LCD is $2 * 2 * 2 * 3 * x * x = 24x^2$	Find the LCD. Build the LCD using the factors above.
	$24x^2 \left(\frac{5}{12x} + \frac{1}{8x^2} \right) = 24x^2 * \frac{1}{3}$ $24x^2 \left(\frac{5}{12x} \right) + 24x^2 \left(\frac{1}{8x^2} \right) = 24x^2 \left(\frac{1}{3} \right)$ $(2x)(5) + (3)(1) = (8x^2)(1)$ $10x + 3 = 8x^2$ $8x^2 - 10x - 3 = 0$ $(4x + 1)(2x - 3) = 0$ $4x + 1 = 0 \text{ or } 2x - 3 = 0$ $x = \frac{-1}{4} \text{ or } x = \frac{3}{2}$	Multiply both sides of the equation by LCD to eliminate (reduce) the fractions. Distribute the LCD to both terms on the left side. Simplify the fractions. This is a 2 nd degree (quadratic) equation. Write the equation in standard form. If possible, factor the trinomial. Set each factor equal to zero. Solve for x . The solutions are in the domain.

Check/Verification	Substitute $x = \frac{-1}{4}$ and $x = \frac{3}{2}$ into the original equation and simplify the results to verify both are solutions.
--------------------	---

4. Solve $\frac{x+3}{x-2} - \frac{4x+7}{x^2+3x-10} = 1$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction, so clear the fractions by multiplying both sides of the equation by the LCD. Since the denominator contains a variable, determine the domain restrictions	
Solution	$x^2 + 3x - 10 = 0$ $x - 2 = 0 \quad \text{and} \quad (x + 5)(x - 2) = 0$ $x = 2 \quad \quad \quad x + 5 = 0 \quad \text{or} \quad x - 2 = 0$ $x = -5 \quad \text{or} \quad x = 2$ <p>Therefore $x \neq 2$ or $x \neq -5$. The domain does not include either value. They cannot be a solution to the equation.</p>	<p>Find the domain.</p> <p>Since there is a polynomial in the denominator factor each denominator. Set each factor equal to zero. Solve for x. State the restrictions to the domain.</p>
	<p>The most number of $x - 2$ factors in either denominator is one, so the LCD must have one $x - 2$. The most number of $x + 5$ factors in either denominator is one, so the LCD must have one $x + 5$.</p> <p>The LCD is $(x + 5)(x - 2)$.</p>	<p>Find the LCD.</p> <p>Build the LCD using the factors above.</p>
	$(x + 5)(x - 2) \left(\frac{x + 3}{x - 2} - \frac{4x + 7}{x^2 + 3x - 10} \right) = (x + 5)(x - 2) * 1$ $(x + 5)(x - 2) \left(\frac{x + 3}{x - 2} \right) - (x + 5)(x - 2) \left(\frac{4x + 7}{x^2 + 3x - 10} \right) = (x + 5)(x - 2)$ $(x + 5)(x + 3) - (4x + 7) = (x + 5)(x - 2)$ $x^2 + 8x + 15 - 4x - 7 = x^2 + 3x - 10$ $x^2 + 4x + 8 = x^2 + 3x - 10$ $x = -18$	<p>Multiply both sides of the equation by LCD to eliminate (reduce) the fractions.</p> <p>Distribute the LCD to both terms on the left side.</p> <p>Simplify the fractions and distribute. Collect like terms.</p> <p>The equation is linear. Solve for x.</p> <p>The solution is in the domain.</p>

Check/Verification	Substitute $x = -18$ into the original equation and simplify the results to verify it is a solution.
--------------------	--

5. Solve $\frac{x^2 + x - 1}{x^2 + 5x + 6} + \frac{1}{x + 3} = \frac{1}{x + 2}$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction, so clear the fractions by multiplying both sides of the equation by the LCD. Since the denominator contains a variable, determine the domain restrictions	
Solution	$x^2 + 5x + 6 = (x + 3)(x + 2)$ $x + 3 \text{ and } x + 2 \text{ are prime.}$ <p>There are only two different factors.</p> $x + 3 = 0 \text{ or } x + 2 = 0$ $x = -3 \text{ or } x = -2$ <p>Thus $x \neq -3$ or $x \neq -2$. The domain does not include either value. They cannot be a solution to the equation.</p>	<p>Find the domain.</p> <p>Since there are polynomials in the denominator, factor each denominator.</p> <p>Set each factor equal to zero.</p> <p>Solve for x.</p> <p>State the restrictions for the domain.</p>
	<p>In factored form, the three denominators are $(x + 3)$, $(x + 2)$, and $(x + 3)(x + 2)$. Therefore the LCD is $(x + 3)(x + 2)$.</p>	<p>Find the LCD.</p> <p>Build the LCD using the factors above.</p>
	$(x + 3)(x + 2) \left(\frac{x^2 + x - 1}{x^2 + 5x + 6} + \frac{1}{x + 3} \right) = (x + 3)(x + 2) \left(\frac{1}{x + 2} \right)$ $(x + 3)(x + 2) \left(\frac{x^2 + x - 1}{x^2 + 5x + 6} \right) + (x + 3)(x + 2) \left(\frac{1}{x + 3} \right)$ $= (x + 3)(x + 2) \left(\frac{1}{x + 2} \right)$ $(x^2 + x - 1) + (x + 2)(1) = (x + 3)(1)$ $x^2 + x - 1 + x + 2 = x + 3$ $x^2 + x - 2 = 0$ $(x + 2)(x - 1) = 0$ $(x + 2) = 0 \text{ or } (x - 1) = 0$ $x = -2 \text{ or } x = 1$	<p>Multiply both sides of the equation by LCD to eliminate (reduce) the fractions.</p> <p>Distribute the LCD to both terms on the left side.</p> <p>Simplify the fractions and distribute.</p> <p>Collect like terms</p> <p>This is a 2nd degree (quadratic) equation.</p> <p>Write the equation in standard form.</p> <p>If possible, factor the trinomial.</p> <p>Set each factor equal to zero.</p> <p>Solve for x. The solution $x = -2$ is not in the domain while $x = 1$ is in the domain.</p>

Check/Verification	<p>$x = -2$ is not a solution to the equation, since $x = -2$ is not in the domain. When $x = -2$ is substituted into the original equation the denominator is zero.</p> <p>The solution $x = 1$ is in the domain. Substitute $x = 1$ into the original equation and simplify the results to verify it is a solution.</p>
--------------------	--

6. Solve $\frac{2x+5}{2x^2+5x-3} + \frac{4}{x^2+5x+6} = \frac{x}{2x^2+3x-2}$.

Identification/Analysis	This is an equation (contains an equal sign). Determine the values for x that make the statement true. This implies you need to isolate x . There is a fraction, so clear the fractions by multiplying both sides of the equation by the LCD. Since the denominator contains a variable, determine the domain restrictions.	
Solution	$2x^2 + 5x - 3 = (2x - 1)(x + 3)$ $x^2 + 5x + 6 = (x + 2)(x + 3)$ $2x^2 + 3x - 2 = (2x - 1)(x + 2)$ <p>There are three different factors. $2x - 1 = 0$ or $x + 3 = 0$ or $x + 2 = 0$ $x = \frac{1}{2}$ or $x = -3$ or $x = -2$</p> <p>Thus $x \neq \frac{1}{2}$, $x \neq -3$ or $x \neq -2$. The domain does not include these values. They cannot be a solution to the equation</p>	<p>Find the domain.</p> <p>Since there are polynomials in the denominator, factor each denominator.</p> <p>Set each different factor equal to zero.</p> <p>Solve for x.</p> <p>State the restrictions for the domain.</p>
	The LCD is $(2x - 1)(x + 3)(x + 2)$	<p>Find the LCD.</p> <p>Build the LCD using the factors above.</p>
	$(2x - 1)(x + 3)(x + 2) \left(\frac{2x + 5}{2x^2 + 5x - 3} + \frac{4}{x^2 + 5x + 6} \right)$ $= (2x - 1)(x + 3)(x + 2) \frac{x}{2x^2 + 3x - 2}$	Multiply both sides of the equation by LCD to eliminate (reduce) the fractions.

<p>Solution continued</p>	$(2x-1)(x+3)(x+2)\left(\frac{2x+5}{2x^2+5x-3}\right) + (2x-1)(x+3)(x+2)\left(\frac{4}{x^2+5x+6}\right)$ $= (2x-1)(x+3)(x+2)\left(\frac{x}{2x^2+3x-2}\right)$ $(x+2)(2x+5) + (2x-1)(4) = (x+3)(x)$ $2x^2 + 5x + 4x + 10 + 8x - 4 = x^2 + 3x$ $x^2 + 14x + 6 = 0$ <p>Find factors that multiply to 6 and add to 14. There is no pair of numbers that does both, so the quadratic does not factor.</p> <p>The quadratic formula states the solutions for the equation</p> $ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ in the above equation}$ <p>$a = 1$, $b = 14$, and $c = 6$ thus</p> $x = \frac{-14 \pm \sqrt{(14)^2 - 4(1)(6)}}{2(1)} = \frac{-14 \pm \sqrt{196 - 24}}{2}$ $= \frac{-14 \pm \sqrt{172}}{2} = \frac{-14 \pm 2\sqrt{43}}{2} = -7 \pm \sqrt{43}$	<p>Distribute the LCD to both terms on the left side.</p> <p>Simplify the fractions and distribute.</p> <p>Collect like terms.</p> <p>This is a 2nd degree (quadratic) equation. Write the equation in standard form. If possible, factor the trinomial.</p> <p>Use the quadratic formula.</p> <p>The solutions are in the domain.</p>
<p>Check/Verification</p>	<p>Substitute $x = -7 + \sqrt{43}$ and $x = -7 - \sqrt{43}$ into the original equation and simplify the results to verify that each is a solution.</p>	

Function Notation:

Function notation replaces the dependent variable y with $f(x)$ where x is the independent variable.

Example: Suppose we have the equation $y = 2x + 7$ and want to know what the value of y is when we replace x with -3 .

Using function notation, the equation would be stated as $f(x) = 2x + 7$ and the question would be stated as: Find $f(-3)$.

Of course the answer is 1, so we would respond $f(-3) = 1$.

1. Given $g(x) = \frac{2x+3}{x-5}$, find $g(4)$.

Identification/Analysis	In this case we have an expression to evaluate. That means substitute 4 in for x and then simplify.	
Solution/evaluate	$g(4) = \frac{2(4)+3}{4-5} = \frac{11}{-1} = -11$	Put 4 in for x and simplify.

2. Find the domain of $g(x) = \frac{2x+3}{x-5}$.

Identification/Analysis	The domain is the set of all possible inputs for the independent variable (in this case x). Since dividing by zero is not possible, we need to set the denominator equal to zero and determine which values of x make the denominator zero.	
Solution	$x - 5 = 0$ $x = 5$ The domain of $g(x)$ is all real numbers except 5. In set-builder notation the domain is written $\{x \mid x \neq 5\}$. In interval notation the domain is written $(-\infty, 5) \cup (5, \infty)$.	Set denominator equal to zero and solve. State the domain.

3. Given $g(x) = \frac{2x+3}{x-5}$, find an x so that $g(x) = 4$.

Identification/Analysis	<p>Now we are asked to find the value(s) for x so that the output or answer is 4.</p> <p>Solve the equation $\frac{2x+3}{x-5} = 4$. This is an equation with a fraction and a variable in the denominator.</p>	
Solution	$x - 5 = 0$ $x = 5$ Thus $x \neq 5$. The domain does not include $x = 5$ so $x = 5$ cannot be a solution to the equation.	Find the domain. Set denominator equal to zero. Solve for x . State the restrictions.
	Since the only denominator is $x - 5$, that is the LCD.	Find the LCD.
	$(x-5) * \frac{2x+3}{x-5} = (x-5) * 4$ $2x+3 = 4x-20$ $-2x = -23$ $x = \frac{23}{2}$	Multiply both sides of the equation by LCD to eliminate (reduce) the fractions. Simplify the fraction and distribute. This is a linear equation. Isolate x . The solution is in the domain.
Check/Verification	$g\left(\frac{23}{2}\right) = \frac{2\left(\frac{23}{2}\right)+3}{\frac{23}{2}-5} = \frac{23+3}{\frac{13}{2}} = 26 \div \frac{13}{2} = 26 * \frac{2}{13} = 4$ <p>Thus $g\left(\frac{23}{2}\right) = 4$ so the solution is $x = \frac{23}{2}$.</p>	