Review of Quadratic Expressions and Equations

A quadratic expression is in the form $ax^2 + bx + c$ where $a \neq 0$.
A quadratic equation is in the form $ax^2 + bx + c = 0$ where $a \neq 0$.

Factoring Quadratic Expressions

One key skill associated with quadratic expressions is factoring. To factor a quadratic expression means write it as a product of linear polynomials. The majority of factoring you will do is factoring quadratic (degree 2) polynomials of three general types.

Type 1: Trinomials of the form $x^2 + bx + c$ where b and c are real numbers.
Type 2: Trinomials of the form $ax^2 + bx + c$ where a, b, and c are real numbers.
Type 3: Binomials of the form $a^2 - b^2$ are called the difference of perfect squares.

Type 1: **Trinomials of the form $x^2 + bx + c$ where b and c are real numbers.**

Find a pair of integers n_1 and n_2 that multiply to c and also add to b. The trinomial will factor as the product of binomials $(x+n_1)(x+n_2)$.

Example: If possible, factor $x^2 + 11x + 24$.

Identification/Analysis	This is a quadratic expression with three terms of the type $x^2 + bx + c$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to all three terms.

Factor

We need to find a pair of integers that multiply to 24 and add to 11. One way to accomplish this is to create a list of all the possible ways to multiply to 24 and then check to see which, if any, add to 11.

Factors of 24	Sum of the pair of factors
$1*24$	25
$2*12$	14
$3*8$	11
$4*6$	10

Notice that $3*8=24$ and $3+8=11$, therefore the trinomial $x^2+11x+24$ factors as $(x+3)(x+8)$.

Check

To check your work, multiply out the answer (distribute) and you should get back to the original trinomial. You may be familiar with the acronym FOIL to remember how to multiply binomials. **F** means multiply the first terms of each binomial together. **O** means multiply the outside terms together; that is the first term of the first binomial multiplied by the second term of the second binomial. **I** means multiply the inside terms together; that is the second term of the first binomial times the first term of the second binomial. Finally, **L** means multiply the last (second) terms of each binomial together. These four products are then added together. So to check our solution $(x+3)(x+8)$, we FOIL.

F	$x*x=x^2$
O	$x*8=8x$
I	$3*x=3x$
L	$3*8=24$

Finally when we add these four terms together, we get $x^2+8x+3x+24=x^2+11x+24$.

Example: If possible, factor $x^2 + 14x - 32$.

Identification/Analysis	This is a quadratic expression with three terms of the type $x^2 + bx + c$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to all three terms.

<table>
<tr>
<td>Factor</td>
<td>We need a pair of integers that multiply to -32 and add to 14. Since the integers must multiply to -32, we know one must be positive and the other negative. Furthermore, since the numbers must add to 14, the larger number (in absolute value) must be the positive one.
<table>
<tr><td>Factors of -32</td><td>Sum of the pair of factors</td></tr>
<tr><td>$-1 * 32$</td><td>31</td></tr>
<tr><td>$-2 * 16$</td><td>14</td></tr>
<tr><td>$-4 * 8$</td><td>4</td></tr>
</table>
This time, the pair of integers that work are $-2 * 16$, so the trinomial $x^2 + 14x - 32$ factors as $(x-2)(x+16)$.</td>
</tr>
</table>

Check	Distribute $(x-2)(x+16)$ and get $x^2 + 16x - 2x - 32$ or $x^2 + 14x - 32$.

Example: If possible, factor $x^2 - 5x - 12$.

Identification/Analysis	This is a quadratic expression with three terms of the type $x^2 + bx + c$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to all three terms.

<table>
<tr>
<td>Factor</td>
<td>We need a pair of integers that multiply to -12 and add to -5. The two numbers we want to find multiply to a negative value, so one must be positive and the other negative. This time, they must add to a negative number, so the larger of the two numbers (in absolute value) must be the negative number.
<table>
<tr><td>Factors of -12</td><td>Sum of the pair of factors</td></tr>
<tr><td>$1 * -12$</td><td>-11</td></tr>
<tr><td>$2 * -6$</td><td>-4</td></tr>
<tr><td>$3 * -4$</td><td>-1</td></tr>
</table>
Notice that none of the pairs of numbers that multiply to -12 also add to -5. This means the trinomial does not factor (trinomial is prime).</td>
</tr>
</table>

Check	Trinomial is prime.

Type 2: **Trinomials of the form ax^2+bx+c where a, b, and c are real numbers. Two methods are shown.**

Method 1:
First we check for a common factor to all three terms. If one exists, factor it out and then proceed; if there is no common factor other than one, continue on. At this point, the strategy is similar to factoring for Type 1, but having a leading coefficient "a" other than one often makes factoring more challenging. With this type of trinomial, we want to find a pair of integers n_1 and n_2 that multiply to ac and also add to b.

Example: If possible, factor $2x^2+3x-20$.

Identification/Analysis	This is a quadratic expression with three terms of the type ax^2+bx+c. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to all three terms.

Factor

We need a pair of integers that multiply to -40 since $2*-20=-40$ and add to 3. This time we need one positive number and one negative number and the larger (in absolute value) of the numbers needs to be positive because the two must add to a positive value.

Factors of -40	Sum of the pair of factors
$-1*40$	39
$-2*20$	18
$-4*10$	6
$-5*8$	3

Since a pair of integers exist that both multiply to -40 and add to 3, the trinomial factors. However, it does not factor as $(x-5)(x+8)$ like those in Type 1 (verify by multiplying). Instead use the two integers to rewrite the middle (bx) term of the trinomial.

$$2x^2+3x-20=2x^2-5x+8x-20$$

Notice the polynomial now has four terms. Next we group the first two terms together and the second two terms together and factor within each group.

$$\left(2x^2-5x\right)+\left(8x-20\right)=x\left(2x-5\right)+4\left(2x-5\right)$$

There are now two terms $x(2x-5)$ and $4(2x-5)$. Notice these two terms have a common factor of $(2x-5)$. Factor it out and we have successfully factored the trinomial.

$$x\left(2x-5\right)+4\left(2x-5\right)=\left(2x-5\right)\left(x+4\right)$$

Check

$$\left(2x-5\right)\left(x+4\right)=2x^2+8x-5x-20=2x^2+3x-20$$

Method 2: Guess and Check

Some students and instructors find the above method too prescriptive or too time consuming and instead favor the guess-and-check method. This alternative is illustrated below with a few guidelines.

Example: If possible, factor $2x^2 + 3x - 20$.

Identification/Analysis	This is a quadratic expression with three terms of the type $ax^2 + bx + c$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to all three terms.

<table>
<tr>
<td>Factor</td>
<td>The first terms of the two binomials must multiply to $2x^2$. There is only one way to multiply to $2x^2$, $2x$ times x. Therefore we know the factors must look like $(2x \pm ?)(x \pm ?)$. Next, the second terms of the two binomials must multiply to -20. That means one of the binomials must have a positive second term and the other a negative second term, so it either factors as $(2x + ?)(x - ?)$ or $(2x - ?)(x + ?)$. We'll guess it factors like $(2x + ?)(x - ?)$ (see Principle B below). Finally, we know the numbers that replace the ? in each binomial must multiply to -20 (notice the signs are already in place to ensure the 20 is negative). Here's where guess-and-check gets its name. Try the factors one pair at a time and check to see if they work. Notice there are two possibilities for each pair of factors because the first terms of the binomials are not the same.
<table>
<tr><th>Pair of factors</th><th>Possible factors</th><th>Check</th></tr>
<tr><td>$1*20$</td><td>$(2x+1)(x-20)$</td><td>$2x^2 -39x-20$</td></tr>
<tr><td>$1*20$</td><td>$(2x+20)(x-1)$</td><td>$2x^2 +18x-20$</td></tr>
<tr><td>$2*10$</td><td>$(2x+2)(x-10)$</td><td>$2x^2 -18x-20$</td></tr>
<tr><td>$2*10$</td><td>$(2x+10)(x-2)$</td><td>$2x^2 +6x-20$</td></tr>
<tr><td>$4*5$</td><td>$(2x+4)(x-5)$</td><td>$2x^2 -6x-20$</td></tr>
<tr><td>$4*5$</td><td>$(2x+5)(x-4)$</td><td>$2x^2 -3x-20$</td></tr>
</table>
</td>
</tr>
</table>

	Remember, at the beginning we decided that one binomial would have the positive value and the other the negative. We just guessed how the signs should go, so in fact the possible factors above are only half of the total possibilities. We could also have all of the above factors with the form $(2x-?)(x+?)$. Fortunately there are two guiding principles that will make this process somewhat simpler.

Principle A: Never even try a possible factoring where one of the binomials would have a common factor. Our very first step was to factor out anything common, so we are guaranteed that no factor of the trinomial will have a common factor. Notice for the example above, we would not need the second, third, fourth, or fifth possibilities because in each of them there is a 2 common to at least one of the binomials. Therefore an observant student would only consider the first and last as possible factors.

Principle B: If you switch the signs in the binomials, the only result is the sign on the bx (middle) term changes.

Factor, continued	For example, the first possibility above is $(2x+1)(x-20)$, with a product of $2x^2-39x-20$. By changing the signs to $(2x-1)(x+20)$, the product is $2x^2+39x-20$. Therefore, the idea to simply guess at the signs is useful. For the given trinomial $2x^2+3x-20$, ideally you get the middle term to be $3x$ when you guess and check, however you should be just as happy getting $-3x$. Notice our last possibility above was $(2x+5)(x-4)$ which is equivalent to $2x^2-3x-20$. It has the "right number" but "wrong sign" compared to the expression we were supposed to factor. All we need to do is switch the signs, and we're done. Therefore, the expression $2x^2+3x-20$ factors as $(2x-5)(x+4)$.

Check	$(2x-5)(x+4)=2x^2+8x-5x-20=2x^2+3x-20$

Example: If possible, factor $6x^2-28x+16$. Guess and Check method illustrated.

Identification/Analysis	This is a quadratic expression with three terms of the type ax^2+bx+c . Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	First, factor out the GCF of 2 which yields $2\left(3x^2-14x+8\right)$.

<table>
<tr>
<td>Factor</td>
<td>
<p>Now proceed with trying to factor $3x^2-14x+8$. I will illustrate the guess and check technique. The first terms of the binomials must be $3x$ and x. Since the last term of the trinomial is positive, the pair of integers that multiply to 8 must either be both positive or both negative. Since the middle term is negative, I know both binomials must have a second term that is negative. Therefore, if the trinomial factors, it looks like $2(3x-?)(x-?)$. Try all the possible pairs of integers that multiply to 8.</p>
<table>
<tr><th>Pair of factors</th><th>Possible factors</th><th>Check</th></tr>
<tr><td>$-1*-8$</td><td>$2(3x-1)(x-8)$</td><td>$2\left(3x^2-25x+8\right)$</td></tr>
<tr><td>$-1*-8$</td><td>$2(3x-8)(x-1)$</td><td>$2\left(3x^2-11x+8\right)$</td></tr>
<tr><td>$-2*-4$</td><td>$2(3x-2)(x-4)$</td><td>$2\left(3x^2-14x+8\right)$</td></tr>
<tr><td>$-2*-4$</td><td>$2(3x-4)(x-2)$</td><td>$2\left(3x^2-10x+8\right)$</td></tr>
</table>
<p>The third option works. Therefore $6x^2-28x+16$ factors as $2(3x-2)(x-4)$.</p>
</td>
</tr>
</table>

Check	$2(3x-2)(x-4)=2\left(3x^2-12x-2x+8\right)=6x^2-28x+16$

Type 3: **Binomials of the form $a^2 - b^2$ are called the difference of perfect squares.**

If an expression is quadratic (degree two) and only has two terms, check to see if it is the difference of two perfect squares. The first term must be positive and the second term negative. Also, each term must be a perfect square (if we ignore the negative sign). If these two conditions are true, then $a^2 - b^2$ factors as the product of $(a+b)(a-b)$.

Example: If possible, factor $x^2 - 16$.

Identification/Analysis	This is a quadratic expression with two terms, called a binomial. Check to see if it is of the form $a^2 - b^2$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to both terms.

Factor	The first term is positive and the second is negative. Further both x^2 and 16 are perfect squares, so this expression is of the form $a^2 - b^2$. Thus $x^2 - 16$ factors as $(x-4)(x+4)$.

Check	$(x-4)(x+4) = x^2 + 4x - 4x - 16 = x^2 - 16$

Example: If possible, factor $49x^2 - 25$.

Identification/Analysis	This is a quadratic expression with two terms. Check to see if it is of the form $a^2 - b^2$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to both terms.

Factor	The first term is positive and the second is negative. Since $49x^2 = (7x)^2$ and $25 = 5^2$, the binomial factors as $(7x+5)(7x-5)$.

Check	$(7x+5)(7x-5) = 49x^2 - 35x + 35x - 25 = 49x^2 - 25$

Example: If possible, factor $-9 + 4z^2$.

Identification/Analysis	This is a quadratic expression with two terms. Check to see if it is of the form $a^2 - b^2$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to both terms.

Factor	At first glance it does not appear to have the correct form. Remember that addition can happen in any order (commutative property). Therefore $-9 + 4z^2$ can be written as $4z^2 - 9$. Now it has the correct form, and both terms are perfect squares. Thus, $4z^2 - 9$ factors as $(2x+3)(2x-3)$.

Check	$(2x+3)(2x-3) = 4x^2 - 6x + 6x - 9 = 4x^2 - 9$

Example: If possible, factor $3x^2 - 36$.

Identification/Analysis	This is a quadratic expression with two terms. Check to see if it is of the form $a^2 - b^2$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is a factor of 3 common to both terms. Factoring out the GCF yields $3\left(x^2 - 12\right)$.

Factor	The remaining binomial $x^2 - 12$ is not a difference of perfect squares because 12 is not a perfect square. Therefore, the binomial will not factor further. The only factoring we were able to do is to take out the GCF.

Check	$3\left(x^2 - 12\right) = 3x^2 - 36$

Example: If possible, factor $x^2 + 36$.

Identification/Analysis	This is a quadratic expression with two terms. Check to see if it is of the form $a^2 - b^2$. Factor means write as the product of two or more polynomials of lower degree.

Check for Common Factor	There is nothing common to both terms.

Factor	Both terms are perfect squares, but neither term is negative. It is not a difference of squares, so it does not factor.

Check	Many students are not convinced this polynomial is not prime. Consider the following possible factorings and see that none of them work. $(x+6)(x+6) = x^2 + 12x + 36$ $(x-6)(x-6) = x^2 - 12x + 36$

	$(x+6)(x-6)=x^2-36$

Solving Quadratic Equations

Zero-product principle:

Suppose two numbers (factors) a and b are multiplied together so that their product equals zero. Then at least one of the numbers (factors) a or b must equal zero.	$a*b=0$ $a=0$ or $b=0$.

Strategy for solving quadratic equations:

Identification/Analysis	Determine if the equation is quadratic. If so, put it in the form $ax^2+bx+c=0$. If not, use the appropriate technique for solving that type of equation.

Check for Common Factor	If there is any common factor in all three terms, factor it out.

Factor	Factor the quadratic expression as one of the three types discussed earlier in this review packet: *Type 1:* Trinomials of the form x^2+bx+c where b and c are real numbers. *Type 2:* Trinomials of the form ax^2+bx+c where a, b, and c are real numbers. *Type 3:* Binomials of the form a^2-b^2 are called the difference of perfect squares. **Note: Not every quadratic will factor. Alternative methods will be discussed in the next section.**

Zero-product principle	Set each factor equal to zero and solve the resulting equation. This will you give you the solutions to the quadratic equation.

Example: Solve the equation $x^2 + 11x + 24 = 0$.

Identification/Analysis	This is a quadratic equation of the type $x^2 + bx + c = 0$

Check for Common Factor	There is nothing common to all three terms.

Factor	We need to find a pair of integers that multiply to 24 and add to 11. Notice that $3 * 8 = 24$ and $3 + 8 = 11$, therefore the trinomial $x^2 + 11x + 24$ factors as $(x+3)(x+8)$.

Zero-product principle	Set each factor equal to zero and solve. $x + 3 = 0$ or $x + 8 = 0$ $x = -3$ or $x = -8$ Therefore the equation $x^2 + 11x + 24 = 0$ has solutions $\{-8, -3\}$.
Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. Check $x = -8$: $\begin{aligned} (-8)^2 + 11(-8) + 24 &= 0 \\ 64 - 88 + 24 &= 0 \\ 0 &= 0 \end{aligned}$ Check $x = -3$: $\begin{aligned} (-3)^2 + 11(-3) + 24 &= 0 \\ 9 - 33 + 24 &= 0 \\ 0 &= 0 \end{aligned}$

Example: Solve the equation $2x^2 - 20 = 6x$.

Identification/Analysis	This is a quadratic equation because the highest degree term is degree 2. However, it is not in the correct form. Write it in the form $ax^2 + bx + c = 0$. $2x^2 - 20 = 6x$ $2x^2 - 6x - 20 = 0$

Check for Common Factor	There is a common factor of 2. $2\left(x^2 - 3x - 10\right) = 0$

Factor	We need to find a pair of integers that multiply to -10 and add to -3. Notice that $2 * (-5) = -10$ and $2 + (-5) = -3$, therefore the trinomial $x^2 - 3x - 10$ factors as $(x+2)(x-5)$, so the trinomial $2x^2 - 6x - 20$ factors as $2(x+2)(x-5)$.

Zero-product principle	Set each factor containing a variable equal to zero and solve. $2 = 0$ is a false statement, so it does not yield any solutions to the equation. $x + 2 = 0$ or $x - 5 = 0$ $x = -2$ or $x = 5$ Therefore the equation $2x^2 - 20 = 6x$ has solutions $\{-2, 5\}$.
Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $2(-2)^2 - 20 = 6(-2)$ Check $x = -2$: $2(4) - 20 = -12$ $-12 = -12$ $2(5)^2 - 20 = 6(5)$ Check $x = 5$: $2(25) - 20 = 30$ $30 = 30$

Example: Solve the equation $x^2 - 49 = 0$

Identification/Analysis	This is a quadratic equation because the highest degree term is degree 2.

Check for Common Factor	There is no common factor.

Factor	This quadratic is of the form $a^2 - b^2$ (difference of squares), so it factors as $(x+7)(x-7)=0$.

Zero-product principle	Set each factor containing a variable equal to zero and solve. $x+7=0$ or $x-7=0$ $x=-7$ or $x=7$ Therefore the equation $x^2 - 49 = 0$ has solutions $\{-7,7\}$.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $(-7)^2 - 49 = 0$ Check $x=-7$: $49 - 49 = 0$ $0 = 0$ $(7)^2 - 49 = 0$ Check $x=7$: $49 - 49 = 0$ $0 = 0$

Example: Solve the equation $2x(x-2)=x+3$.

Identification/Analysis	This is a quadratic equation because the highest degree term is degree 2. However, it is not in the correct form. Write it in the form $ax^2+bx+c=0$. $2x(x-2)=x+3$ $2x^2-4x-x-3=0$ $2x^2-5x-3=0$

Check for Common Factor	There is no common factor.

Factor	Using which ever strategy for factoring trinomials of the type ax^2+bx+c that works best for you, verify the trinomial factors as $(2x+1)(x-3)=0$.

Zero-product principle	Set each factor containing a variable equal to zero and solve. $2x+1=0$ or $x-3=0$ $x=-\frac{1}{2}$ or $x=3$ Therefore the equation $2x(x-2)=x+3$ has solutions $\left\{-\frac{1}{2},3\right\}$.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $2\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-2\right)=-\frac{1}{2}+3$ Check $x=-\frac{1}{2}$: $2\left(-\frac{1}{2}\right)\left(-\frac{5}{2}\right)=\frac{5}{2}$ $\frac{5}{2}=\frac{5}{2}$ $2(3)(3-2)=3+3$ Check $x=3$: $2(3)(1)=6$ $6=6$

Example: Solve the equation $(x-7)(x-1)=-8$.

Identification/Analysis	This is a quadratic equation because the highest degree term is degree 2. However, it is not in the correct form. Write it in the form $ax^2+bx+c=0$. $(x-7)(x-1)=-8$ $x^2-x-7x+7=-8$ $x^2-8x+15=0$

Check for Common Factor	There is no common factor.

Factor	The numbers that multiply to 15 and add to -8 are -3 and -5, so the trinomial factors as $(x-3)(x-5)=0$.

Zero-product principle	Set each factor containing a variable equal to zero and solve. $x-3=0$ or $x-5=0$ $x=3$ or $x=5$ Therefore the equation $(x-7)(x-1)=-8$ has solutions $\{3,5\}$.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $(3-7)(3-1)=-8$ Check $x=3$: $(-4)(2)=-8$ $-8=-8$ $(5-7)(5-1)=-8$ Check $x=5$: $(-2)(4)=-8$ $-8=-8$

Quadratic Formula:

The solutions of the quadratic equation $ax^2+bx+c=0$ are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

The quadratic formula will solve every quadratic equation.

If $b^2-4ac>0$, the equation has two, distinct real solutions.
If $b^2-4ac=0$, the equation has one real solution (called a double root).
If $b^2-4ac<0$, the equation has two complex solutions.

Example: Solve the equation $x^2-5x-12=0$.

Identification/Analysis	This is a quadratic equation of the type $x^2+bx+c=0$

Check for Common Factor	There is nothing common to all three terms.

Factor	We need to find a pair of integers that multiply to -12 and add to -5. There are no two numbers that do this. Therefore the trinomial is prime. We must use the quadratic formula to solve the equation.

Quadratic formula	To use the quadratic formula, $a=1$, $b=-5$, $c=-12$. $x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(-12)}}{2(1)}$ Then $=\dfrac{5\pm\sqrt{25+48}}{2}$ $=\dfrac{5\pm\sqrt{73}}{2}$

	Therefore the solutions to the equation $x^2 - 5x - 12 = 0$ are $\frac{5+\sqrt{73}}{2}$ and $\frac{5-\sqrt{73}}{2}$.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $\left(\frac{5+\sqrt{73}}{2}\right)^2 - 5\left(\frac{5+\sqrt{73}}{2}\right) - 12 = 0$ Check $x = \frac{5+\sqrt{73}}{2}$: $\frac{25+10\sqrt{73}+73}{4} - \frac{25+5\sqrt{73}}{2} - 12 = 0$ $\frac{98+10\sqrt{73}}{4} - \frac{50+10\sqrt{73}}{4} - \frac{48}{4} = 0$ $0 = 0$ $\left(\frac{5-\sqrt{73}}{2}\right)^2 - 5\left(\frac{5-\sqrt{73}}{2}\right) - 12 = 0$ Check $x = \frac{5-\sqrt{73}}{2}$: $\frac{25-10\sqrt{73}+73}{4} - \frac{25-5\sqrt{73}}{2} - 12 = 0$ $\frac{98-10\sqrt{73}}{4} - \frac{50-10\sqrt{73}}{4} - \frac{48}{4} = 0$ $0 = 0$

Example: Even if the quadratic equation can be solved by factoring, the quadratic formula will work. Consider one of our previous examples: Solve the equation $x^2+11x+24=0$.

Quadratic formula	To use the quadratic formula, $a=1$, $b=11$, $c = 24$. $x=\frac{-(11)\pm\sqrt{(11)^2-4(1)(24)}}{2(1)}$ $=\frac{-11\pm\sqrt{121-96}}{2}$ Then $=\frac{-11\pm\sqrt{25}}{2}$ $=\frac{-11\pm5}{2}$ $=\frac{-11+5}{2}$ or $\frac{-11-5}{2}$ $=-3$ or -8 Therefore the solutions to the equation $x^2+11x+24=0$ are -8 and -3.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $(-8)^2+11(-8)+24=0$ Check $x=-8$: $64-88+24=0$ $0=0$ $(-3)^2+11(-3)+24=0$ Check $x=-3$: $9-33+24=0$ $0=0$

Example: Solve the equation $2x^2 = -x - 7$.

Identification/Analysis	This is a quadratic equation because the highest degree term is degree 2. However, it is not in the correct form. Write it in the form $ax^2 + bx + c = 0$. $2x^2 + x + 7 = 0$

Check for Common Factor	There is nothing common to all three terms.

Factor	We need to find a pair of integers that multiply to 14 and add to1. There are no two numbers that do this. Therefore the trinomial is prime. We must use the quadratic formula to solve the equation.

Quadratic formula	To use the quadratic formula, $a = 2$, $b = 1$, $c = 7$. $x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(2)(7)}}{2(2)}$ Then $= \dfrac{-1 \pm \sqrt{1-56}}{4}$ $= \dfrac{-1 \pm \sqrt{-55}}{4}$ These are complex solutions because the number under the radical is negative. We write $\sqrt{-55}$ as $\sqrt{-1} \cdot \sqrt{55} = i\sqrt{55}$. Therefore the solutions to the equation $2x^2 = -x - 7$ are $\dfrac{-1 + i\sqrt{55}}{4}$ and $\dfrac{-1 - i\sqrt{55}}{4}$.

Check	To check the answers, substitute each answer into the original equation and be sure it yields a true statement. $2\left(\frac{-1+i\sqrt{55}}{4}\right)^2 = -\left(\frac{-1+i\sqrt{55}}{4}\right)-7$ $2\left(\frac{1-2i\sqrt{55}+55i^2}{16}\right) = \frac{1-i\sqrt{55}}{4}-7$ Check $x = \frac{-1+i\sqrt{55}}{4}$: $\frac{1-2i\sqrt{55}-55}{8} = \frac{2-2i\sqrt{55}}{8}-\frac{56}{8}$ $\frac{-2i\sqrt{55}-54}{8} = \frac{-2i\sqrt{55}-54}{8}$ $0 = 0$ $2\left(\frac{-1-i\sqrt{55}}{4}\right)^2 = -\left(\frac{-1-i\sqrt{55}}{4}\right)-7$ $2\left(\frac{1+2i\sqrt{55}+55i^2}{16}\right) = \frac{1+i\sqrt{55}}{4}-7$ Check $x = \frac{-1-i\sqrt{55}}{4}$: $\frac{1+2i\sqrt{55}-55}{8} = \frac{2+2i\sqrt{55}}{8}-\frac{56}{8}$ $\frac{2i\sqrt{55}-54}{8} = \frac{2i\sqrt{55}-54}{8}$ $0 = 0$

Key characteristics of the graph of a quadratic expression.

Identification/Analysis	The graph of $f(x) = ax^2 + bx + c$ is called a parabola

Open up or down	The parabola opens up if $a > 0$ and opens down if $a < 0$.

$y -$ intercept	To find the $y -$ intercept of the parabola, let $x = 0$ and solve for y. Notice the $y -$ intercept is always $(0, c)$.

$x -$ intercept(s)	To find the $x -$ intercept(s) of the parabola, let $y = 0$ and solve for x. This results in a quadratic equation to solve. You may get zero, one, or two $x -$ intercepts.

vertex	The vertex of the parabola is the high point (maximum) if the parabola opens down and the low point (minimum) if the parabola opens up. The vertex has its x coordinate at $x = -\frac{b}{2a}$ and the y coordinate is found by plugging $x = -\frac{b}{2a}$ into the equation $f(x) = ax^2 + bx + c$. Therefore the vertex of the parabola is at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Example: Discuss the graph of the quadratic $y = 3x^2 + 11x + 10$.

Identification/Analysis	The graph of $y = 3x^2 + 11x + 10$ is a parabola

Open up or down	The parabola opens up because $a = 3$ which is positive.

$y -$ intercept	To find the $y -$ intercept of the parabola, let $x = 0$ and solve for y. $y = 3(0)^2 + 11(0) + 10$ $y = 10$ The $y -$ intercept is located at $(0,10)$.

$x -$ intercept(s)	To find the $x -$ intercept(s) of the parabola, let $y = 0$ and solve for x. $3x^2 + 11x + 10 = 0$ $(3x + 5)(x + 2) = 0$ $3x + 5 = 0$ or $x + 2 = 0$ $x = -\frac{5}{3}$ or $x = -2$ Therefore the $x -$ intercept(s) are located at $\left(-\frac{5}{3},0\right)$ and $(-2,0)$.

vertex	Since this parabola opens up, the vertex is the lowest point (minimum) on the graph. The x coordinate is $x = -\frac{11}{2(3)} = -\frac{11}{6}$. $y = 3\left(-\frac{11}{6}\right)^2 + 11\left(-\frac{11}{6}\right) + 10$ $= 3\left(\frac{121}{36}\right) + 11\left(-\frac{11}{6}\right) + 10$ The y coordinate is $= \frac{121}{12} - \frac{121}{6} + 10$ $= \frac{121}{12} - \frac{242}{12} + \frac{120}{12}$ $= -\frac{1}{12}$ Therefore the vertex of the parabola is located at $\left(-\frac{11}{6}, -\frac{1}{12}\right)$.